12.1 The interaction of matter with radiation

- Equations
    - Planck relationship: $E = hf$
    - Einstein photoelectric equation: $E_{max} = hf - \Phi$
    - Bohr orbit energies: $E = -\frac{13.6\,eV}{n^2}$
    - Quantization of angular momentum: $mvr = \frac{nh}{2\pi}$
    - Probability density: $P(r) = |\psi|^2 \Delta V$
    - Heisenberg relationship: position-momentum $\Delta x \Delta p \geq \frac{h}{4\pi}$
    - Energy-Time: $\Delta E \Delta t \geq \frac{h}{4\pi}$
- The photoelectric effect
    - Demonstration of the photoelectric effect
        - The photoelectric effect can be demonstrated using the gold-leaf electroscope.
            - In this experiment, a clean sheet of zinc should be mounted on electroscope plate and the sheet charged negatively by connecting to a high negative potential.
                - When radiation from infrared to UV is shown, the leaf will only collapse back down when UV light is shown on the negative plate.
                - This is because of the photoelectric effect.
    - Explanation of the photoelectric effect
        - Einstein explained the photoelectric effect in the following ways:
            - Light consists of particles known as photons.
                - The energy of an individual photon is given by: $E = hf$
                - The total energy of the photon is given by: $E = Nhf$
            - Each photon interacts with a single electron.
            - There is a minimum light frequency, called "threshold frequency (f_0)", below which no electron is emitted.
            - Energy is needed to overcome the attractive forces that act on the electron within the metal
                - This energy is given by the quantity known as the work function (Φ).
            - Any further energy supplied to the electron will become kinetic energy of the emitted electron.
                - The emitted electron is often called a photoelectron.
            - Increasing the intensity of light simply increases the number of photons incident per second.
                - They won't increase the energy of the photoelectrons.
        - Explaining the gold leaf experiment
            - The zinc sheet has a certain work function to free the electrons on the sheet.
                - The photons incident on the zinc sheet must have at least the same amount of energy, if not more than the work function to free the electrons.
                - UV light has the highest frequency, meaning that it will be able to free the electrons.
                - When any intense visible or infra-red light on the sheet the leaf remains diverged
                    - Increasing the intensity of the light while maintaining a low frequency will emit 0 electrons.
                        - This is because the photons won't overcome the Φ.
                        - The intensity will only increase the number of photons incident per-second.
                - If low intensity ultraviolet is used, the leaf falls immediately
                    - Electrons are instantly expelled from the metal.
                        - Meaning the zinc plate is no longer negatively charged.
                        - Causing the electrostatic forces of repulsion to collapse, and the leaf to fall down.
                - Placing a sheet of glass between the UV light source and the zinc prevents the leaf from falling
                    - Glass will only transmit low energy visible photons, while absorbing high energy UV ones.
                - If the zinc sheet is charged positively the leaf remains diverged for all wavelengths of radiation
                    - Raising the potential of the plate will mean that the force of attraction between the plate and electrons will increase.
                        - This means that the work function increases as well.
                        - It's said that the potential well will increase.

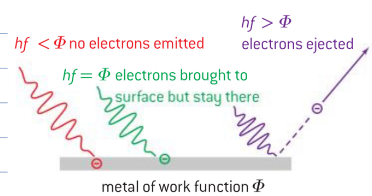


Figure 3 Photoelectric emission and the work function.

- Einstein's photoelectric equation
    - Einstein expressed the photoelectric effect with the following formula:
        - $E_{max} = hf - \Phi$
            - In this equation "E_{max}" is the maximum kinetic energy of the emitted electrons.

- h is plancks constant ($6.63 \cdot 10^{-34}$)
- f is the frequency of light.
- Φ is the work function of the metal.

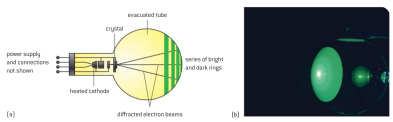
- Everything in the equation is measured in joules.
  - although, they can also be expressed in electron volts.
- The energy is written as a maximum because, the work function is the minimum energy required to free an electron.
  - Electrons further into the metal will require more energy to free.

- Worked example
  - $\Phi = 6.6 \cdot 10^{-19} J$
    - $f_0 = \frac{\Phi}{h}$
      $= \frac{6.6 \cdot 10^{-19}}{6.63 \cdot 10^{-34}}$
      $= 1 \cdot 10^{15} Hz$
    - $E_{max} = hf - \Phi$
      $= (6.63 \cdot 10^{-34})(1.2 \cdot 10^{15}) - (6.6 \cdot 10^{-19})$
      $= 1.4 \cdot 10^{-19} J$
  - $E = 2.2 eV$, $\Phi = 1.5 eV$
    $(2.2 \cdot 1.6 \cdot 10^{-19}) = hf$
    $f = \frac{(2.2 \cdot 1.6 \cdot 10^{-19})}{6.63 \cdot 10^{-34}}$
    $= 5.3 \cdot 10^{14} Hz$
  - $E_{max} = (5.3 \cdot 10^{14})(6.63 \cdot 10^{-34}) - (1.5 \cdot 1.6 \cdot 10^{-19})$
    $= 1.1 \cdot 10^{-19} J$
    $1.1 \cdot 10^{-19} = \frac{1}{2} mv^2$
    $v = 4.9 \cdot 10^{5} ms^{-1}$

- The wave theory and the photoelectric effect
  - the reason that wave theory for light fails to explain the photoelectric effect is because of the instantaneous nature of electron expulsion.
    - its proven that there is no time delay from when light is incident on a metal surface to when an electron is freed.
    - since waves provide a continuous supply of energy, and the intensity of a wave is proportional to the squared amplitude. then with sufficient time, electrons will be expelled, which isnt the case.
      - this is because according to classic wave theory, when low intensity electromagnetic radiation of any frequency, given enough time, and an electron would be freed from the potential well.
      - However this obviously isnt the case as only photons with a frequency above the threshold frequency will free electrons without time delay.
        - Below this frequency, no electrons are emitted, completely contradicting wave theory
  - since light still has wave like properties, these being diffraction and interference, then its said to have wave-particle duality.
  - Worked example
    - $f = 1.2 \cdot 10^{15} Hz$, $\Phi = 1.8 eV$
      - $E = hf$
        $= (6.63 \cdot 10^{-34})(1.2 \cdot 10^{15})$
        $= 8.0 \cdot 10^{-19} J$
      - $E_{max} = hf - \Phi$
        $= 8.0 \cdot 10^{-19} - (1.8 \cdot 1.6 \cdot 10^{-19})$
        $= 5.12 \cdot 10^{-19} J = 3.2 eV$
      - $E_{max} = eV_s$
        $V_s = \frac{5.12 \cdot 10^{-19}}{1.6 \cdot 10^{-19}}$
        $= 3.2V$

- Matter waves
  - De Broglie used the idea of light acting as a wave and a particle to come up with the following idea:
    - If something is classically thought to act as a wave, then a species that is thought to be a particle must have wave-like properties.
    - this brought him to the "de Broglie wavelength":
      - $\lambda = \frac{h}{p}$
      - where h is the planck constant, and p is the momentum of the species.
  - The total energy of any object is its kinetic energy plus its rest energy, shown in the formula below:
    - $E^2 = c^2p^2 + m_0^2c^4$
    - since photons dont have a rest mass, their total energy is given by:
      - $E = cp$
  - the de Broglie wavelength can be derived from the photo electric effect and the total energy equation:
    - since $E = cp$ and $E = hf = \frac{hc}{\lambda}$ equating them will give:
      - $cp = \frac{hc}{\lambda}$
        $\lambda = \frac{h}{p}$

- Electron diffraction
  - The de Broglie wavelength was experimentally demonstrated by showing interference maxima when a beam of electrons was reflected by a nickel crystal.
  - Electrons from a heated cathode, pass through a thin film of carbon atoms (crystal).
    - If the electrons acted as particles, then they would only be slightly deviated by collisions with the carbon atoms.



▲ Figure 6 Electron diffraction tube.

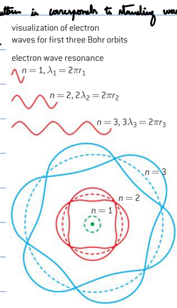
- this would mean that they would form a bright region at the centre of the screen.
- the bright rings on the image are where the electrons landed.
  - the bright regions represent high probabilities of electrons reaching that point while dark areas represent low probabilities.
  - the same pattern is made, no matter the amount of electrons incident per second.
    - this pattern is very similar to that of the interference pattern of light with a diffraction grating.
    - showing that electrons also behave as waves.
- As electrons are accelerated through a potential difference, they gain kinetic energy.
  - $eV = \frac{1}{2}mv^2$ ($E_k = \frac{1}{2}mv^2$)
  - Assuming that the electrons don't move at the speed of light, their momentum will equal $p = mv$, meaning $p^2 = (mv)^2$
    - so $\frac{1}{2}mv^2 = \frac{p^2}{2m} = eV$.
    - meaning $p = \sqrt{2meV} = \sqrt{2mE_k}$
    - Using de Broglie's wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

- Worked example
  - Electron accelerated through pd of 3kV.
    - $\lambda = \frac{h}{\sqrt{2meV}}$

      $\lambda = \frac{(6.63 \cdot 10^{-34})}{\sqrt{2(9.11 \cdot 10^{-31} \cdot 1.6 \cdot 10^{-19} \cdot 3000)}}$

      $\lambda = 2.24 \cdot 10^{-11}$ m
- the diffraction angle (θ) is reduced in the equation $n\lambda = d\sin\theta$ when the voltage is increased.
  - this is because the wavelength decreases as the voltage increases, mean the electron diffracts less.

- the Bohr model
  - the Bohr model (where electrons have fixed amounts of energy), made 3 key assumptions:
    - Electrons in an atom exist in stationary states.
      - Electrons can remain in this state without emitting any electromagnetic radiation.
    - Electrons may move from one stationary state to another by emitting or absorbing a quantum (fixed/exact) of electromagnetic radiation.
      - If an electron orbiting a nucleus absorbs a photon, it can move up to a higher "excited" state.
        - the difference between the two energy levels must be equal to the energy of the photon.
        - If an excited electron moves to a lower energy state, it will emit a photon.
          - It will be equal to the energy difference of the two energy levels.
      - the difference in the energy levels is given by the equation: $\Delta E = hf$.
    - The angular momentum of an electron in a stationary state is quantised in integral values of $\frac{h}{2\pi}$
      - this can be represented mathematically by:
        - $mvr = \frac{nh}{2\pi}$
      - Angular momentum is the (vector) product of the momentum of a particle and the radius of its orbit.
        - therefore, for a particle in a circular orbit, the angular momentum will be constant.
      - the pattern is corresponds to standing waves.
        -

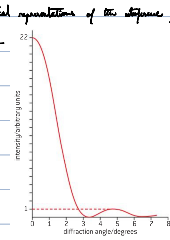


▲ Figure 7 Standing waves in an atom for $n = 2$ and $n = 3$.

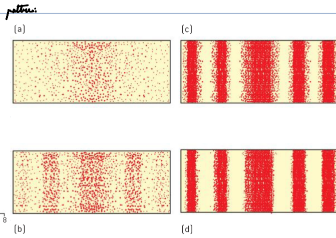
- Energies in the Bohr orbits
  - In the hydrogen atom, the total kinetic and potential energy of the energy levels can be represented by the equation:
    - $E = -\frac{13.6}{n^2}$
    - n is the energy states, where n=1 is the ground state, and n=∞ is equal to the ionisation/infinite energy level.
    - the energy is negative because the electron is in a potential well.
      - Due to the attractive forces of the positive nucleus on the electron.

- **Worked example**
    - $mvr = \frac{nh}{2\pi}$
    - When an electron moves from a higher energy level to a lower one, it will release a photon of the same frequency and wavelength that is found in the hydrogen emission spectrum.

- **Schrödinger's equation**
    - Wave-particle duality explains the bright fringes as being areas where there is a high probability of finding a particle.
        - These probability waves superpose with one another to produce the expected interference pattern.
        - With this pattern, we can determine where we're likely to find an electron, but we're not able to determine its actual location.
        - Diagrammatical representations of the interference pattern:

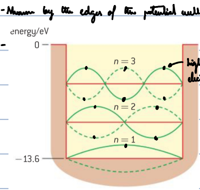


▲ Figure 9 Diffraction pattern intensity distribution.



▲ Figure 10 Diffraction pattern being built up by individual electrons.

    - Schrödinger's wave function "ψ" describes the quantum state of particles.
        - For a light wave, we observe that $I \propto \chi^2$.
        - The wave function shows where we're most likely to find a photon.
            - For the wave function, where the square of the amplitude is a maximum, there is the greatest probability of finding a photon.
            - When the wave function is zero, then the probability of finding a photon is zero.
    - The square of the amplitude of the wave function $|\Psi|^2$ is proportional to the probability per unit volume of finding the particle.
        - This is known as the probability density: $P(r) = |\psi|^2 \Delta V$
            - $P(r)$ is the probability of finding a particle a distance r from an origin, and ΔV is the volume considered.
    - For the double slit diffraction, in terms of probability waves, we assume that the single photon or electron is everywhere until its observed on the screen.
        - This is known as the Copenhagen interpretation.
        - The Copenhagen Interpretation can be summed up to "nothing is real unless its observed".
            - Meaning that only the properties of the species measured at that point in time are what are definite.
    - In the simplified version of the hydrogen atom, an electron can would be detected somewhere between the nucleus and the outside edge of the atom.
        - Shown by the edges of the potential well.



▲ Figure 12 Electron standing waves in a potential well.

        - The potential varies as the inverse of the distance from the nucleus: $V \propto -\frac{1}{r}$.
        - In the electron wave model, the probability of finding an electron inside the nucleus or outside the atom are both zero.
            - As the wave amplitude is zero.
            - The electron has the highest probability of being found half way between the nodes.
    - **Worked example**
        - The electron is most likely to be found at "a" as $|\Psi|^2$ is a maximum.
            - There is no chance of finding the electron above "a" or the nucleus.

- **The Heisenberg uncertainty principle**
    - When a quantum is diffracted, then its path can only be put in terms of a probability wave.
        - This outcome falls in line with the Heisenberg uncertainty principle.
    - The Heisenberg uncertainty principle is written as $\Delta x \Delta p \geq \frac{h}{4\pi}$.
        - The uncertainty principle puts a limit on how precisely we can know the position (Δx) and momentum (Δp) of an electron or photon.
        - If Δx is very small, then Δp will be very large, and vice versa.
            - Therefore, its impossible to know the precise position or momentum of an electron.
        - If we imagine an electron that is free from any electric field, then we can graph it as a sine wave since we know its wavelength perfectly.

- This means we will know its momentum perfectly ($p = \frac{h}{\lambda}$).
- This means that the uncertainty in the position is infinite, and we expect the electron to be spread out over all of space.
- When an electron is diffracted, the uncertainty in position will be given by half of the slit.

- Worked example
  - C

- Pair production and pair annihilation
  - Close to an atomic nucleus, where the electric field is very strong, a photon with the right amount of energy can turn into a particle & its antiparticle.
    - This process is known as pair production.
    - The "right amount" of energy means that the energy of the photon must equal the rest mass of the particle and antiparticle combined.
      - This is given by the equation: $E = 2mc^2$
        - "m" is the rest mass of the particle. This value is the same as the antiparticle.
    - The reason that they're produced in pairs is to conserve charge, lepton number, baryon number, and strangeness.
  - For the pair production of a positron and electron, a gamma ray must have energy equal to 1.02 MeV.
    - Any excess photon energy is turned into kinetic energy for the positron and electron.

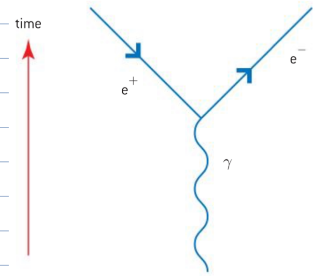


Figure 13 Feynman diagram of electron–positron pair production.

  - Pair production can also occur near an orbiting electron.
    - Although in this case, more energy will be required, as some of the energy will be given to the electron in orbit.
    - This is shown in the following figure:
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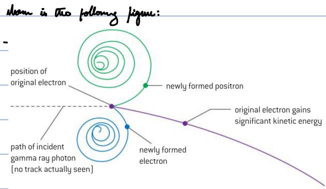


Figure 14 Bubble chamber tracks of electron–positron pair production.

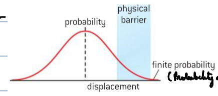
      - As seen, the gamma ray (photon) doesn't have a trail, while the produced electron and positron do.
        - Even the original electron has a trail.
        - The recoiling electron gains kinetic energy, meaning that it won't bend that much in a magnetic field.
        - It was shown that this type of pair production requires $4mc^2 = 2.04\,MeV$.
    - The equation is: $\gamma + e^- \longrightarrow e^- + e^- + e^+$
  - When a particle meets its antiparticle, they will annihilate.
    - This produce a photon with the energy equal to the rest mass of the two particles plus their kinetic energies.

- Pair production and the Heisenberg uncertainty principle
  - Instead of position and momentum in the Heisenberg uncertainty principle, energy and time are used.
    - They're known as conjugate variables.
    - This equation is written as: $\Delta E \Delta t \geq \frac{h}{4\pi}$.
  - What is interesting is that the threshold energy required for the productions of an electron-positron pair can be less than the expected 1.02 MeV when the photon is near a heavy nucleus.
    - If a 10 eV photon is near a heavy nucleus, then the low energy photon will produce an electron-positron pair.
    - A very short time later, the electron and positron collide to produce two photons of 5 eV.
      - This is allowed due to the uncertainty principle.
      - During the lifetime of the electron-positron pair, there is an uncertainty regarding the energy.
        - If this uncertainty was equal to 1.02 MeV, then we can determine the lifetime of the pair.
        - $\Delta t = \frac{h}{4\pi(10^6 \cdot 1.6 \cdot 10^{-19} \cdot 1.2)} = 3.2 \cdot 10^{-22}\,s$

      - Since the time is so short, we can't detect it occurring.
        - If we can't detect it, then quantum mechanics says that there's a chance of it occurring.

- Quantum Tunneling
  - According to quantum mechanics, a particle's wave function has a finite probability of being everywhere in the universe at the same time.
    - It may be infinitesimally small, but it's finite nonetheless.
    - This means that an electron in the ground state of hydrogen can escape the attraction of the nucleus with less than 13.6 eV.
      - A particle can essentially "borrow" energy from its surrounding, pass through a barrier, and then release the energy.
        - This can be done as long as the time is short.

- This is represented by the following graph:



(Probability of quantum tunneling occurring)

▲ Figure 15 Quantum tunnelling to pass through a physical barrier.

- Due to quantum tunneling, the sun can fuse together two hydrogens at a relatively low temperature.
  - To overcome the electrostatic forces of repulsion between the two nuclei the hydrogens would have to have 1MeV of energy.
  - This translates to a core temp of 10^{10} K, which in reality is $3 \cdot 10^{7}$ K.
    - Due to the high pressure and small probability of fusion occurring with quantum tunneling, fusion can occur at lower temperatures.
    - Due to the huge number of Hydrogen nuclei in the sun, a low probability still translates to a lot of Helium being produced.
- Worked example
  - The wave functions refer to Schrödingers wave function $|\Psi|$, where the square of it shows the likelyhood of finding an electron in that position.
  - $\frac{2 \cdot 10^{-10}}{6} = \underline{3.3 \cdot 10^{-11} m}$ - $\Delta p = \frac{h}{4\pi \cdot 2 \cdot 10^{-10}}$
    $p = \frac{h}{\lambda}$ $= \underline{2.6 \cdot 10^{-25} Ns}$
    $p = \underline{2.0 \cdot 10^{-23} Ns}$

12.2 Nuclear physics

- Equation
  - Relationship between radius of nucleus and nucleon number: $R = R_0 A^{\frac{1}{3}}$
  - Decay equation for number of nuclei at time t: $N = N_0 e^{-\lambda t}$
  - Decay equation for activity at time t: $A = \lambda N_0 e^{-\lambda t}$
  - Angle of electron diffraction first minimum: $\sin\theta \approx \frac{\lambda}{D}$
- Rutherford scattering and the nuclear radius
  - The method of closest approach
    - The alpha particles that were back scattered in the gold foil experiment hit the nucleus head on.
      - Only 1 in 8000 alpha particles were back scattered.
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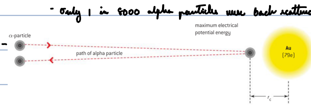
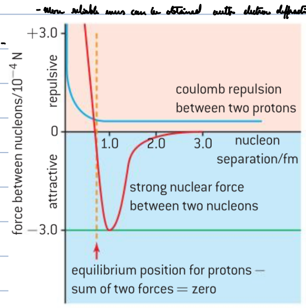


▲ Figure 1 Method of closest approach.

  - In the image above, as the alpha particle moves in closer to the atom, it will lose kinetic energy and instead gain electrical potential energy.
    - When the alpha particle has moved to the closest point to the nucleus, its kinetic energy is zero and has stopped moving for a second.
    - At that point (distance "r_c" from the nucleus), equating kinetic energy to electric potential energy gives:
      - $E_\alpha = \frac{kZe \cdot 2e}{r_c}$ Alpha particle charge
      - k is the coulomb constant, Z is the proton number of gold, and E_α is the kinetic energy of the alpha particle.
      - The equation is an approximation because the gold nucleus is considered to be a point mass.
    - If the alpha particle were to have penetrated the nucleus, then the coulomb constants wont apply.
      - This is because the strong nuclear force is dominant within a nucleus.
  - Rutherford was able to determine that the radius of an atom could be given by the expression:
    - $R = R_0 A^{\frac{1}{3}}$
    - Rutherford was able to come up with this equation by knowing that $V \propto A$ (volume is proportional to nucleon number), and the nuclear radius $R \propto A^{\frac{1}{3}}$.
- Nuclear density
  - If the nucleus is a sphere the volume can be calculated using the equation:
    - $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi A R_0^3$
  - The density of nuclear material will be given by:
    - $\rho = \frac{m}{V} = \frac{Au}{\frac{4}{3}\pi A R_0^3} = \frac{3u}{4\pi R_0^3}$
      - u is the uniform atomic mass, and Au is the total mass of a nucleus of nucleon number A.
      - The equation shows that the density of the nuclei is independent of the nucleons.
        - This is because the density only dependent on constants.
        - $\rho = \frac{3(1.66 \cdot 10^{-27})}{4\pi(1.2 \cdot 10^{-15})^3} = 2.3 \cdot 10^{17}\ kg\,m^{-3}$
          - In nature the only space with this density is a neutron star.
- Deviations from the Rutherford model

- The scattering experiments were limited by energies of the alpha particles emitted by the radioactive sources available to them.
    - When the experiment is done with a more energetic alpha particle the Rutherford scattering relationship doesn't agree.
    - At higher energies the alpha particles can go so close to the nucleus of the atom that the strong force will overcome the electrostatic forces of repulsion.
- The closest approach method tells us the approx. size of a nucleus.
    - More reliable ones can be obtained with electron diffraction.



▲ Figure 2 Variation of the strong nuclear force and coulomb force with distance.

- **Electron diffraction**
    - As electrons are leptons and not hadrons, they will not be affected by the strong nuclear force.
        - Although, they are affected by the charge distribution of the nucleus.
    - For light incident on a small round object of diameter D, the angle θ that the first diffraction minimum makes with the straight through position (θ=0°) is given by:
        - $\sin\theta = \frac{\lambda}{D}$
        - The formula is the same for electron diffraction.
            - where D is the nuclear diameter, and λ is the de Broglie wavelength of the electron.
            - To achieve an appropriate de Broglie wavelength, a 400 MeV electron is used.
                - The wavelength of an electron is: $\lambda = \frac{hc}{E}$
    - **Worked example**
        - R of Calcium-40 is 4.54 fm, E = 420 MeV
            - $R = R_0 A^{\frac{1}{3}}$
              $= (1.2\cdot10^{-15})(40)^{\frac{1}{3}}$
              $= \underline{4.1\cdot10^{-15}\,m}$
        - $\lambda = \frac{hc}{E}$
          $= \underline{2.9\cdot10^{-15}\,m}$
        - $\sin\theta = \frac{\lambda}{D}$
          $= \frac{(2.9\cdot10^{-15})}{9.08\cdot10^{-15}}$
          $= \underline{18.6^{0}}$
        - $\lambda = \frac{hc}{E}$
          $= \frac{(6.63\cdot10^{-34})(3\cdot10^{8})}{(50\cdot10^{6}\cdot1.6\cdot10^{-19})}$ $\frac{(2.5\cdot10^{-15})}{9.08\cdot10^{-15}} \times \sin\theta$
          $\lambda = \underline{2.5\cdot10^{-14}\,m}$
- **Using electrons of higher energies**
    - When electrons with a greater energy than 420 MeV are used in the scattering experiment, then the collision will no longer be elastic.
        - The bombarding electrons will lose kinetic energy.
        - This energy is "converted" into the mass of several mesons when the electron interacts with the nucleus.
    - At even higher energies, the electron will penetrate further into the nucleus causing the quarks to be scattered.
        - This is known as deep inelastic scattering.
        - This also provides direct evidence of the quark model of a nucleus.
    - **Energy levels in a nucleus**
        - Much of the evidence of energy levels in a nucleus comes from radioactive decay.
        - The emission of $^{4}_{2}\alpha$ or $^{0}_{-1}\beta^{-}$/$^{0}_{+1}\beta^{+}$ radiation usually leaves the daughter nuclei in an excited state.
            - To go to the ground state, the nucleus will undergo gamma radiation.
        - For example, when a americium-241 decays to neptunium-237, it can undergo multiple decay paths to reach the ground state.
            - The initial decay will be alpha.
                - Although, the alpha particle will have different energies

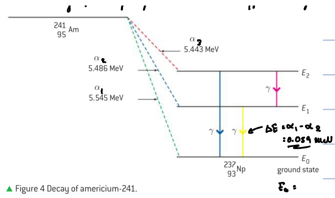


Figure 4 Decay of americium-241.

- As seen in the graph, there are 4 different paths that the particle could decay to the ground state.
- Furthermore, what is also seen is that the energy levels are quantised.
  - Providing evidence of nuclear energy levels.
- The method in which the alpha particle leaves the nucleus is more complex than emitting a gamma ray.
  - Alpha particles will be formed in the nucleus.
    - Meaning two protons and two neutrons will form a cluster.
  - The alpha particles will have a kinetic energy which is less than that required to escape the nucleus.
    - Although, similarly to how hydrogen atoms fuse together in the sun with significantly less energy than required to overcome the electrostatic forces of repulsion, alpha particles can be expelled from the nucleus.
      - As long as the time is very short, the uncertainty in the energy can do it.
  - What has been determined is that the longer the potential barrier is, and larger nucleus, the longer the half life of the nucleus.

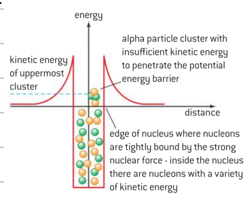


Figure 5 Classical mechanics view of alpha decay.

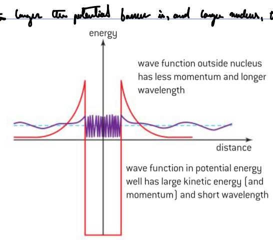


Figure 6 Quantum tunnelling view of alpha decay.

- When the wave function is at a maximum, then that's the highest chance of tunnelling.
  - Meaning that alpha particles with specific energies will be more likely to tunnel.

- **Negative Beta decay**
  - An antineutrino will accompany the emitted electron by the β^-.
  - Experiments show that β^- decay has a continuous energy spectrum.
    - On the other hand, gamma and alpha decay is quantised.

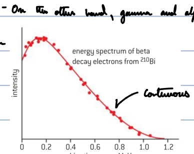


Figure 7 Negative beta-energy spectrum.

- This proves the existence of the antineutrino, because it would be able to conserve the energy if they're added together.

- **The law of radioactive decay**
  - Radioactive decay is a completely random and unpredictable process.
    - what we do know is that the more radioactive nuclei there are, the likelier the chance of radioactive decay occuring.
  - The decay constant (λ) is defined as: the probability that an individual nucleus will decay in a given time interval.
  - The activity of a sample (A) is defined as the number of nuclei decaying in a second.
    - The units are Bq (becquerel).
  - The activity of N undecayed nuclei is given by the equation:
    - $A = \lambda N$
    - The equation will become: $A = \lambda N_0 e^{-\lambda t}$
      - As This equation can be used to determine the exponential decay of the radioactive particle.

- **Worked example**
  - $2200 = \lambda N_0 e^{-\lambda t}$
    $N_0 = \frac{2200}{1.35 \cdot 10^{-11}}$
    $N_0 = 1.63 \cdot 10^{14}$ nuclei

    $n = \frac{N}{N_A}$
    $n = 2.71 \cdot 10^{-10}$ mol

    $226 g\,mol^{-1} \cdot 2.71 \cdot 10^{-10}$
    $= 6.1 \cdot 10^{-8} g$

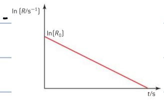
- **Decay constant and half-life**
    - The half-life is defined as the time taken for the number of radioactive nuclei to decrease by half.
        - Therefore, $N = \frac{N_0}{2}$.
        - The equation $N = N_0 e^{-\lambda t}$ can be used as $t_{\frac{1}{2}} = \frac{0.693}{\lambda}$.
- **Measuring long half-lives**
    -


Figure 9 Graph of natural log of the corrected count rate against time.

    - When a sample that has a very long half life needs to have its decay constant measured, the following process is used:
        - Measure the mass of the sample.
        - Calculate the rate.
        - The activity is then determined by:
            - $A = \frac{\text{count rate} \times \text{area of sphere facing the G-M counter}}{\text{area of G-M tube window}}$